Matlab in Signal Processing

Mario Huemer

JOHANNES KEPLER
UNIVERSITY LINZ

INSTITUTE OF SIGNAL PROCESSING

Content

- **Matlab in teaching**
  - Examples of the Bachelor lecture *Signal Processing*
  - Examples of the Master lecture *Statistical and Adaptive Signal Processing*
- **Matlab in research**
  - Example: Short Range Leakage Cancellation in Automotive Radar MMICs
  - Example: Self Interference Cancellation in Mobile Phone Transceiver RFICs
  - Example: UW-OFDM

Digital Signal Processing
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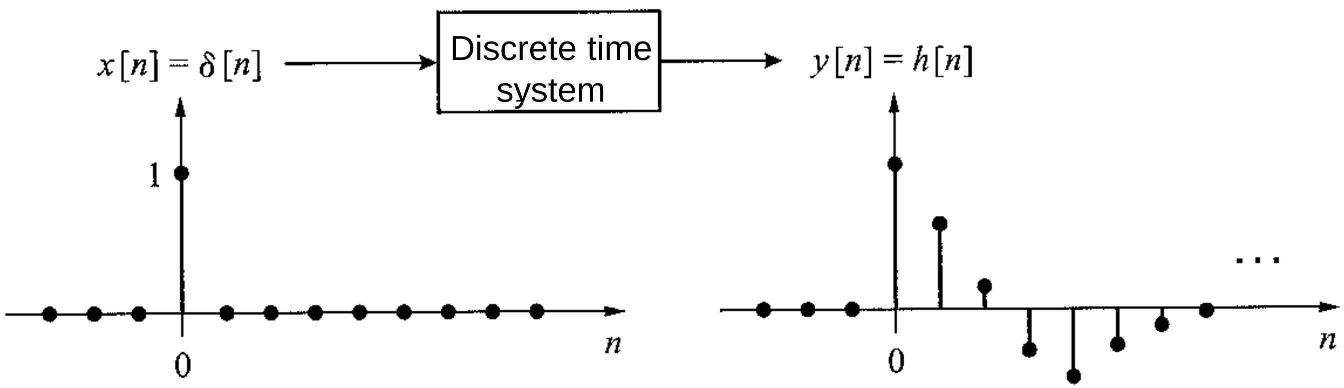
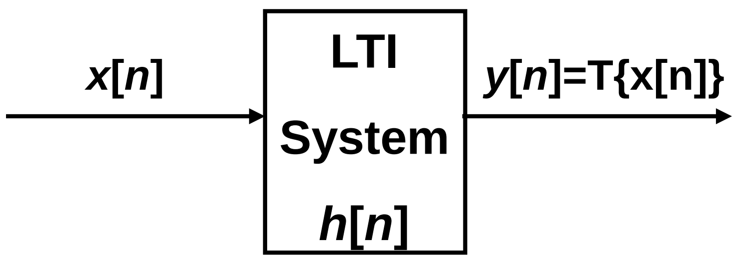
Discrete Time LTI-Systems - Fundamentals

- **Impulse response $h[n]$ of a discrete time LTI system**
  The impulse response of a discrete time LTI system is the reaction $y[n] := h[n]$ of the system to the unit impulse $x[n] = \delta[n]$.



- In general, the following is true for the input-output operator $T\{x[n]\}$:



$$y[n] = T\{x[n]\} = \sum_{i=-\infty}^{\infty} x[i]h[n-i]$$

…discrete convolution

- **Example:** Moving Average Filter

Output signal: $$y[n] = \frac{1}{M+1} \sum_{k=0}^{M} x[n-k]$$

This system calculates the nth sample of the output sequence as the average value of $x[n], x[n-1], \dots, x[n-M]$.

Impulse response: $$h[n] = \frac{1}{M+1} \sum_{k=0}^{M} \delta[n-k]$$

$$= \begin{cases} \frac{1}{M+1} & \text{for } 0 \le n \le M \\ 0 & \text{else} \end{cases}$$

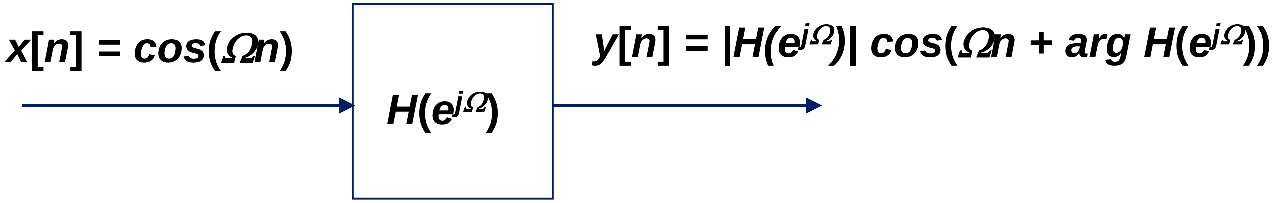
Discrete Time LTI-Systems
Frequency Response

- **Frequency response of an LTI-system**
  - The term

$$H(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\Omega n} \qquad \text{für} \quad -\infty < \Omega < \infty$$

is called *frequency response* of the system.

- Real valued LTI systems feature the typical property that a sinusoidal input signal is processed into a sinusoidal output signal:



The sinusoidal output signal experiences a modification of the amplitude with the factor $\left|H\left(e^{j\Omega}\right)\right|$ and a phase shift $\arg\left(H\left(e^{j\Omega}\right)\right)$ with respect to the input.

Spectral Analysis
Fourier Series – Real Representation

JKU

- A periodic signal $x(t)$ with the period T_0 can be decomposed into a Fourier series.

Fourier series:

$$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty}\left[a_k \cos(2\pi k f_0 t) + b_k \sin(2\pi k f_0 t)\right]$$

Periodic functions can be expressed as a linear combination of sine and cosine oscillations.

Fourier coefficients a_k, b_k:
= spectral representation

$$a_k = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\cos(2\pi k f_0 t)\,dt$$

$$b_k = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\sin(2\pi k f_0 t)\,dt$$

- $f_0 = 1/T_0$ is the frequency of the fundamental oscillation.
- Oscillations of the frequency $f = kf_0$ are called the k^{th} harmonic.
  - $k = 1$: first harmonic or fundamental oscillation ($f = f_0$)
  - $k = 2$ (n): second (n^{th}) harmonic with $f = 2f_0$ ($f = nf_0$)
  - $a_0/2$: constant component (DC component)

Spectral Analysis
Fourier Series – Complex Representation

- Example: By replacing the cosine and sine waves with the Euler formulas

$$\cos(2\pi k f_0 t) = \frac{1}{2}\left(e^{j2\pi k f_0 t} + e^{-j2\pi k f_0 t}\right)$$
$$\sin(2\pi k f_0 t) = \frac{1}{2j}\left(e^{j2\pi k f_0 t} - e^{-j2\pi k f_0 t}\right)$$

we obtain the mathematically most elegant form of the Fourier series:

Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty}\left[c_k e^{j2\pi k f_0 t}\right]$$

Complex Fourier coefficients c_k
= spectral representation

$$c_k = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t) e^{-j2\pi k f_0 t} dt$$

Discrete spectrum = line spectrum:
The spectrum of the periodic signal is defined at the discrete (equidistant) frequencies $k \cdot f_0$.

- A periodic function can therefore also be understood as a weighted sum of complex oscillations.

- As with the real representation, integration must be carried out over one period. The starting time of the integration is irrelevant.

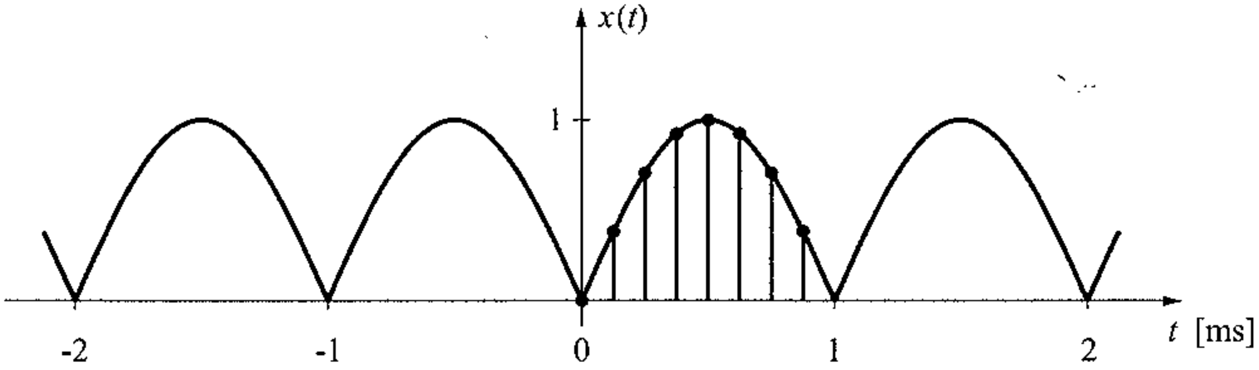
The DFT as an Approximation
of the Fourier Series

- Assuming that exactly one period of a periodic signal is sampled, the Fourier coefficients can be determined according to:

$$c_k \approx \frac{1}{N} X[k] \quad \text{for } k = -N/2, \ldots, N/2 - 1$$

(holds for even *N*, similarly for odd *N*)

$X[k]$ ...DFT coefficients



- Equality applies in the above formula if the periodic signal is band-limited and if the sampling theorem is fulfilled.

Spectral Analysis
Fourier Series – Examples

Example: Derive the Fourier series coefficients of the signal

$$x(t) = 2 + 6\cos(2\pi f_0 t) + 4\sin(2\pi \cdot 2 f_0 t) + 2\cos(2\pi \cdot 3 f_0 t)$$

($f_0 = 1\text{kHz}$) with Matlab.

Example: Derive the Fourier series coefficients of the signal

$$x(t) = |\sin(2\pi \cdot 2 f_0 t)|$$

($f_0 = 1\text{kHz}$) with Matlab.

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Example: Sinusoidal Parameter Estimation

Assume the data to be

$$x[n] = A\cos(\Omega n + \varphi) + w[n] \qquad n = 0,1,\ldots, N-1$$

with $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I})$, $A > 0$ and $0 < \Omega < \pi$. A, Ω, φ shall be estimated.

- FFT-based solution:

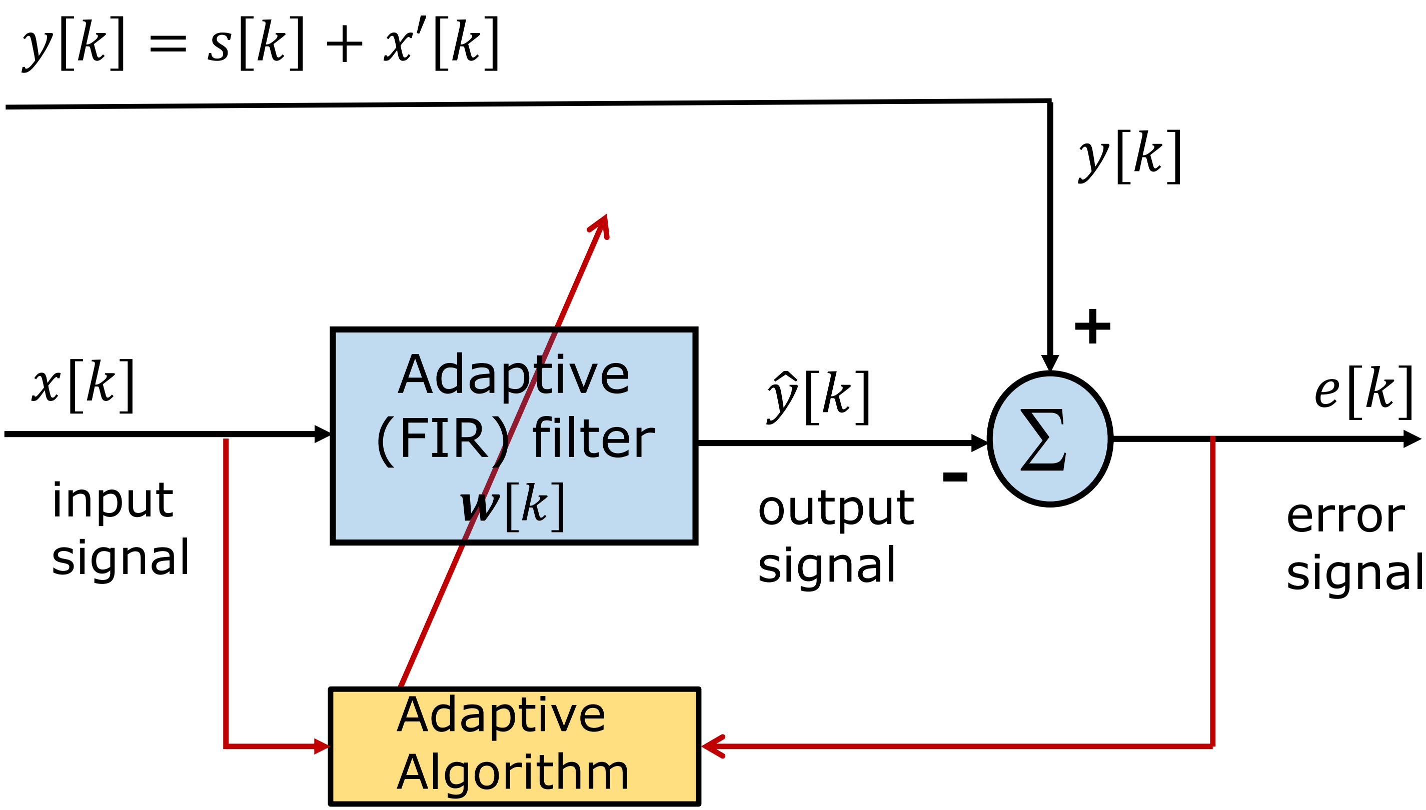
$$\hat{\Omega} = \arg\max \frac{1}{N} \left| \sum_{n=0}^{N-1} x[n] \exp(-j\Omega n) \right|^2$$

$$\hat{A} = \frac{2}{N} \left| \sum_{n=0}^{N-1} x[n] \exp(-j\hat{\Omega} n) \right|$$

$$\hat{\varphi} = \arctan \frac{-\sum_{n=0}^{N-1} x[n] \sin(\hat{\Omega} n)}{\sum_{n=0}^{N-1} x[n] \cos(\hat{\Omega} n)}$$

Adaptive Filters

Adaptive filtering example:



$y[k]$ could, e.g., be a measured ECG signal disturbed by a strong 50Hz interference, then it makes sense to feed an adaptive filter with $x[k] = \cos(2\pi \cdot 50\text{Hz}\, t)$ (see Matlab example).

Example: **Vehicle Tracking**

- *Assumptions: Vehicle moving at constant speed, but perturbed by wind gusts, slight speed corrections, etc., as might occur in an aircraft. The measurements are noisy versions of the range and bearing.*
- *Tracking with Extended Kalman Filter*

Rudolf Kalman, Kurt Schlacher and Mario Huemer at EUROCAST 2013 in Gran Canaria

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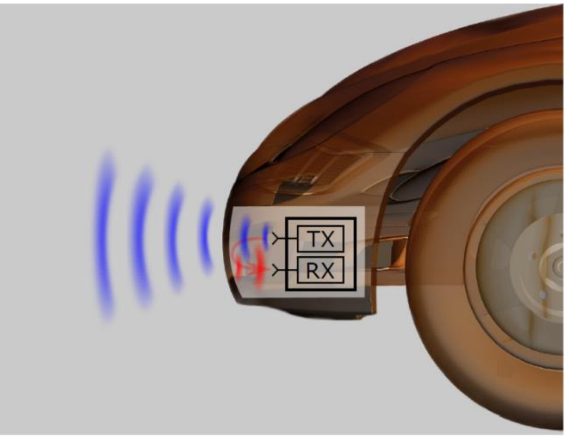
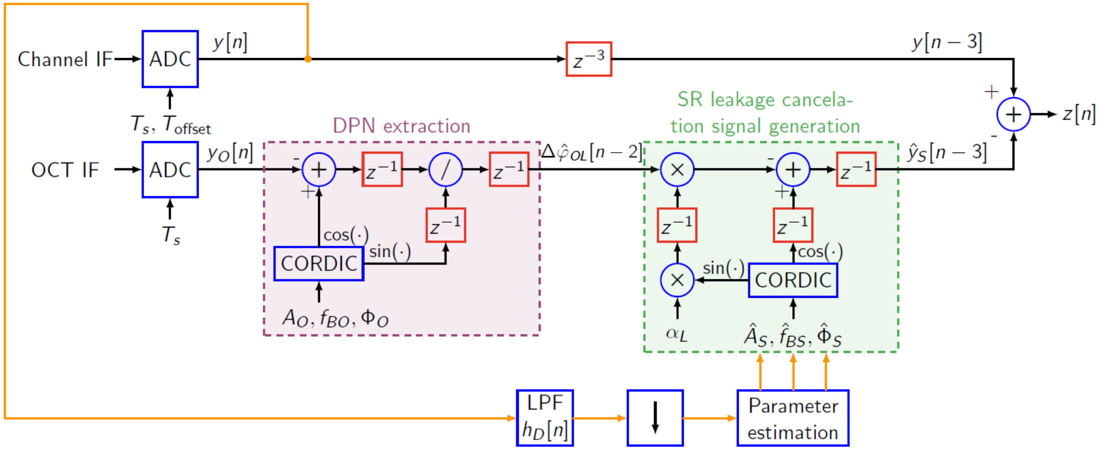
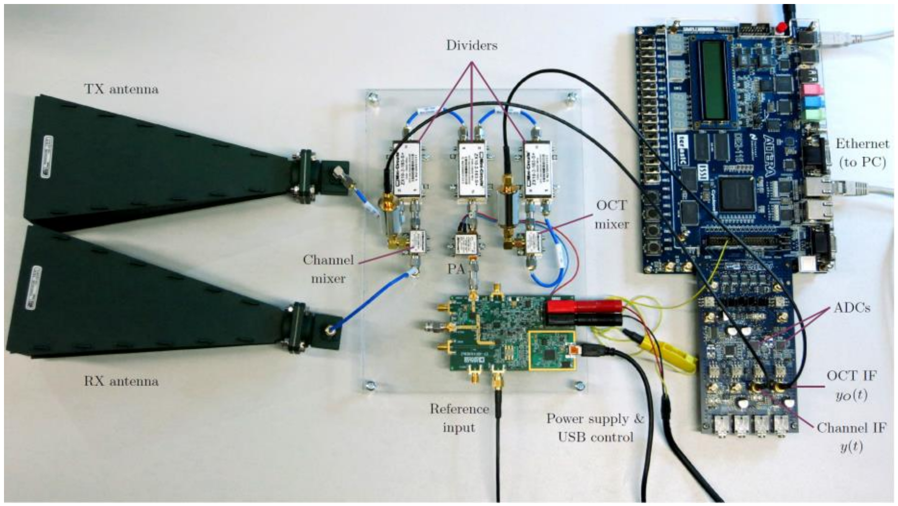
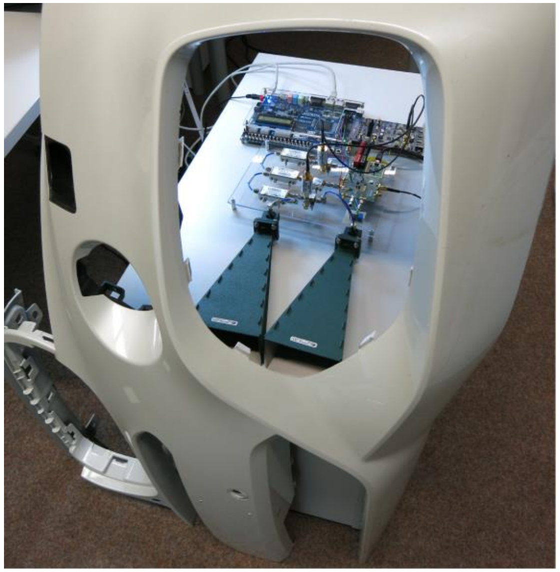
Short-Range Leakage Cancellation in FMCW Radar Transceivers (with Infineon)

- Goal: Cancellation of interspersed signals from narrow targets with emphasis on Phase Noise



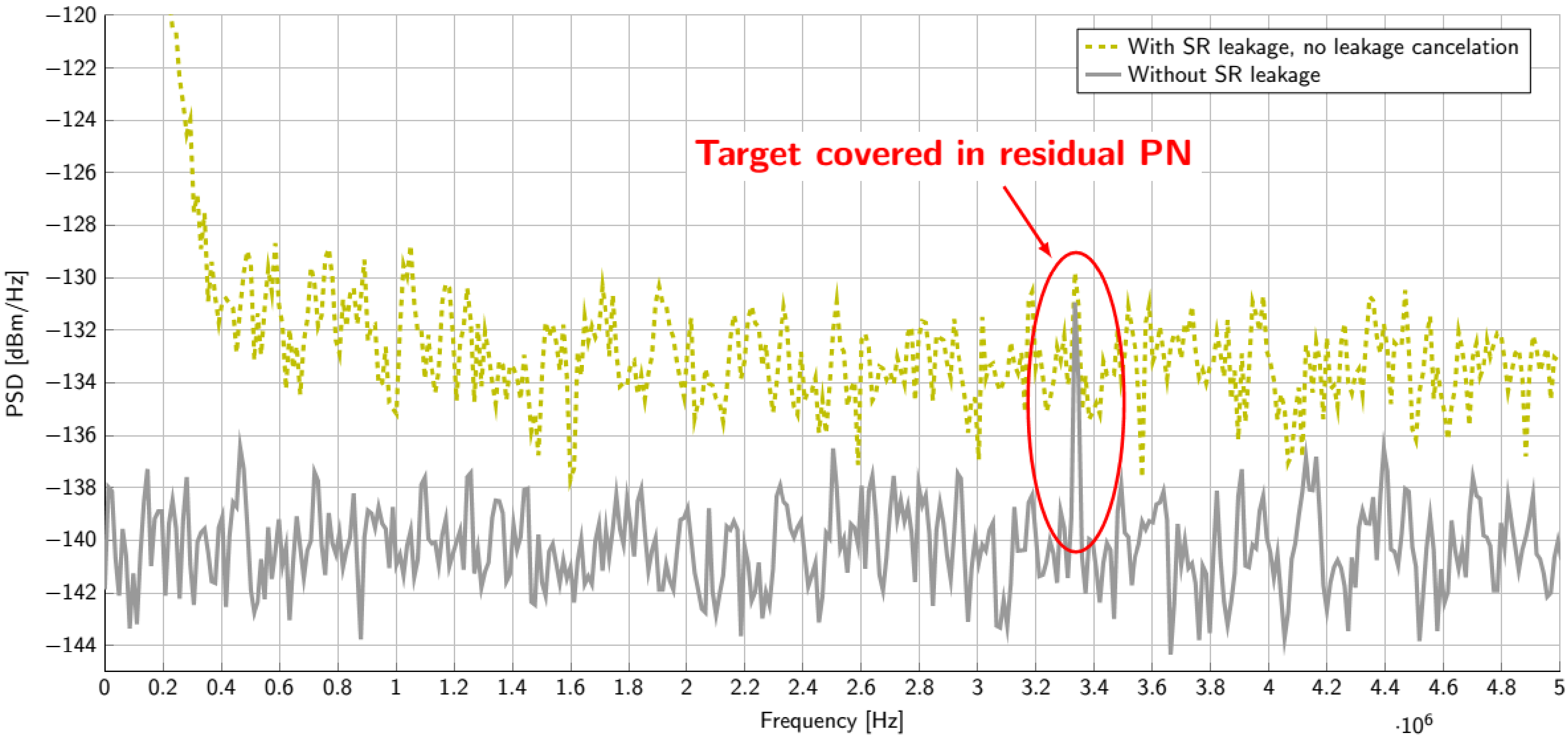






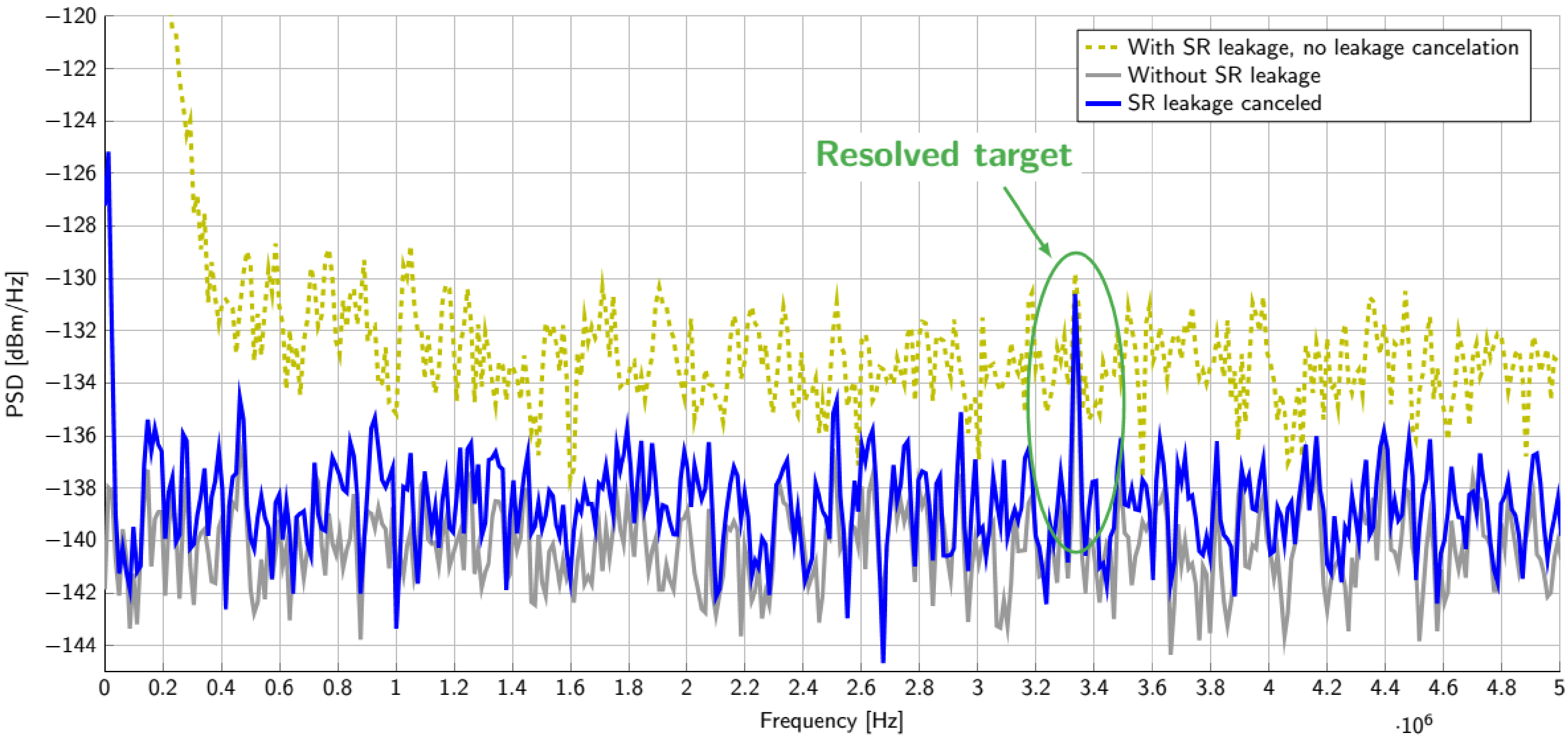
Short-Range Leakage Cancellation in FMCW Radar Transceivers (with Infineon)

- Residual PN of SR leakage increases overall system noise floor, thus degrading detection sensitivity by approx. 6 dB
- Target within channel is covered in noise



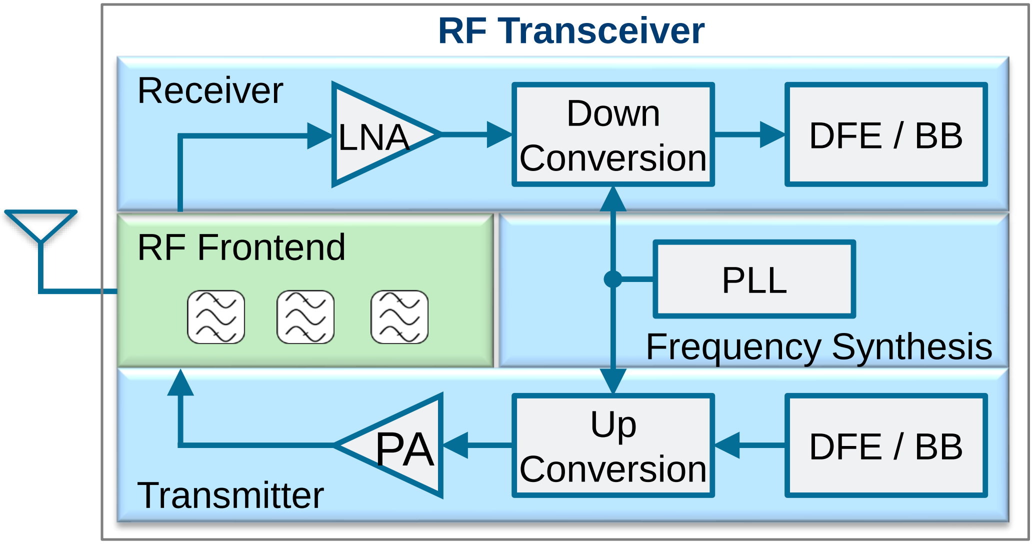
Short-Range Leakage Cancellation in FMCW Radar Transceivers (with Infineon)

- Employing SR leakage cancelation significantly improves target detection sensitivity
- Target within channel is resolved well



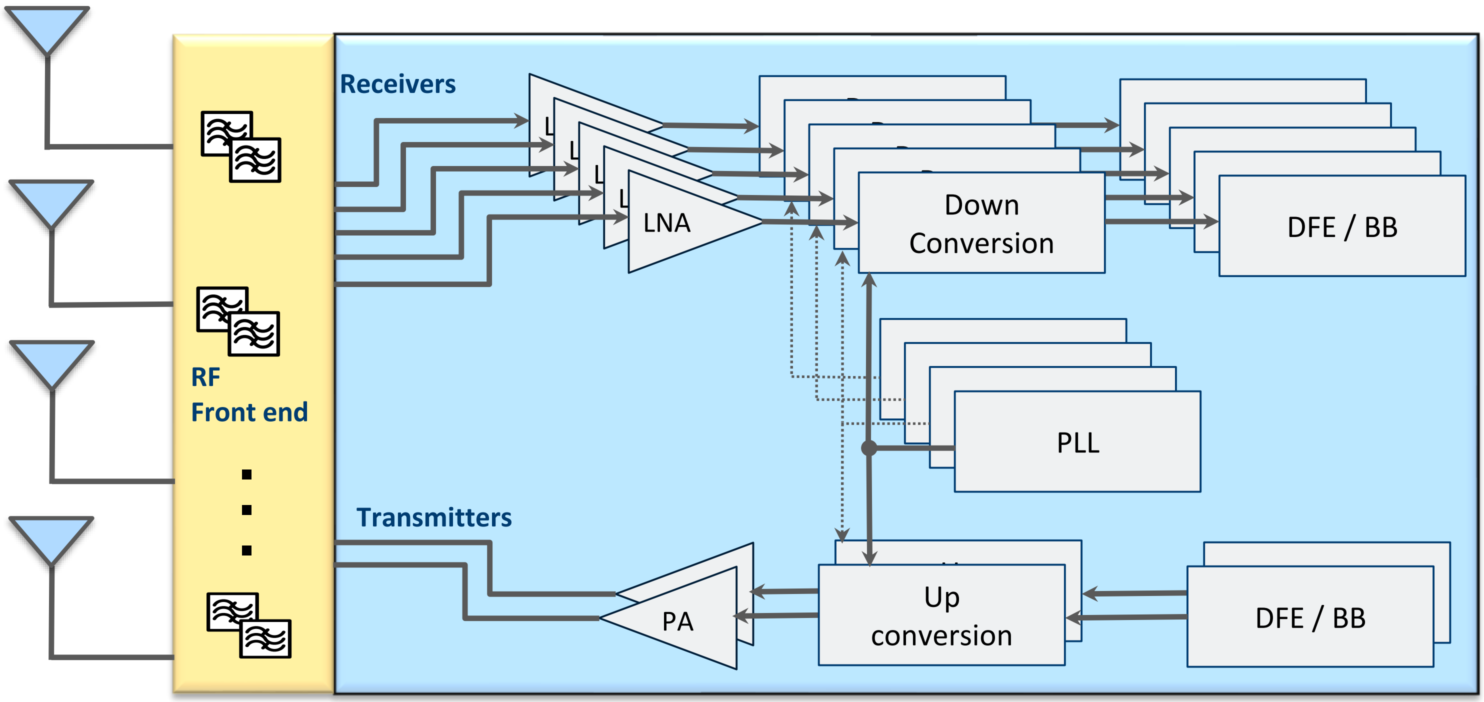
Self Interference Cancellation in Mobile Phone Transceiver RFICs (with Apple)

- Main components of RF transceiver
  - Transmitter
  - Receiver
  - RF front-end

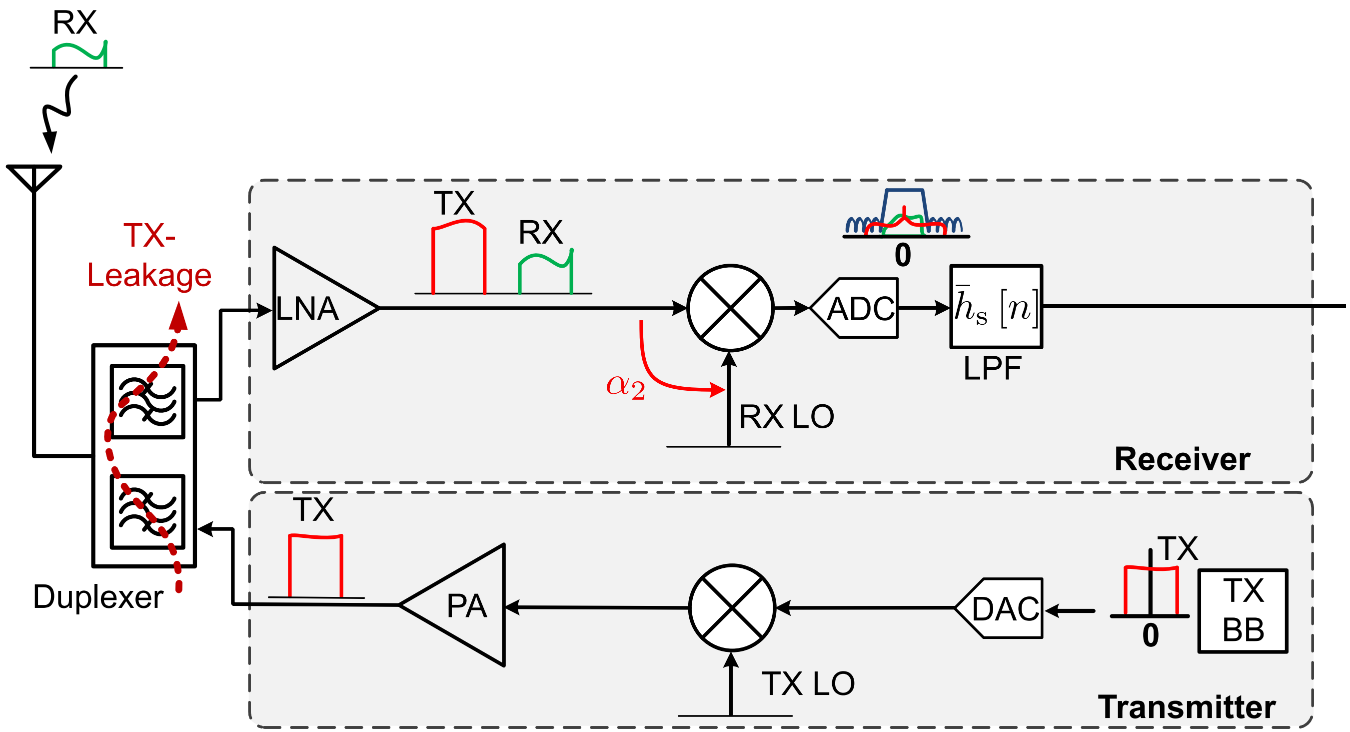


Self Interference Cancellation in Mobile Phone Transceiver RFICs (with Apple)

- Higher data rates → higher bandwidths
- Carrier aggregation (CA) → multiple Rx / Tx chains
- More bands → increased RF front-end complexity

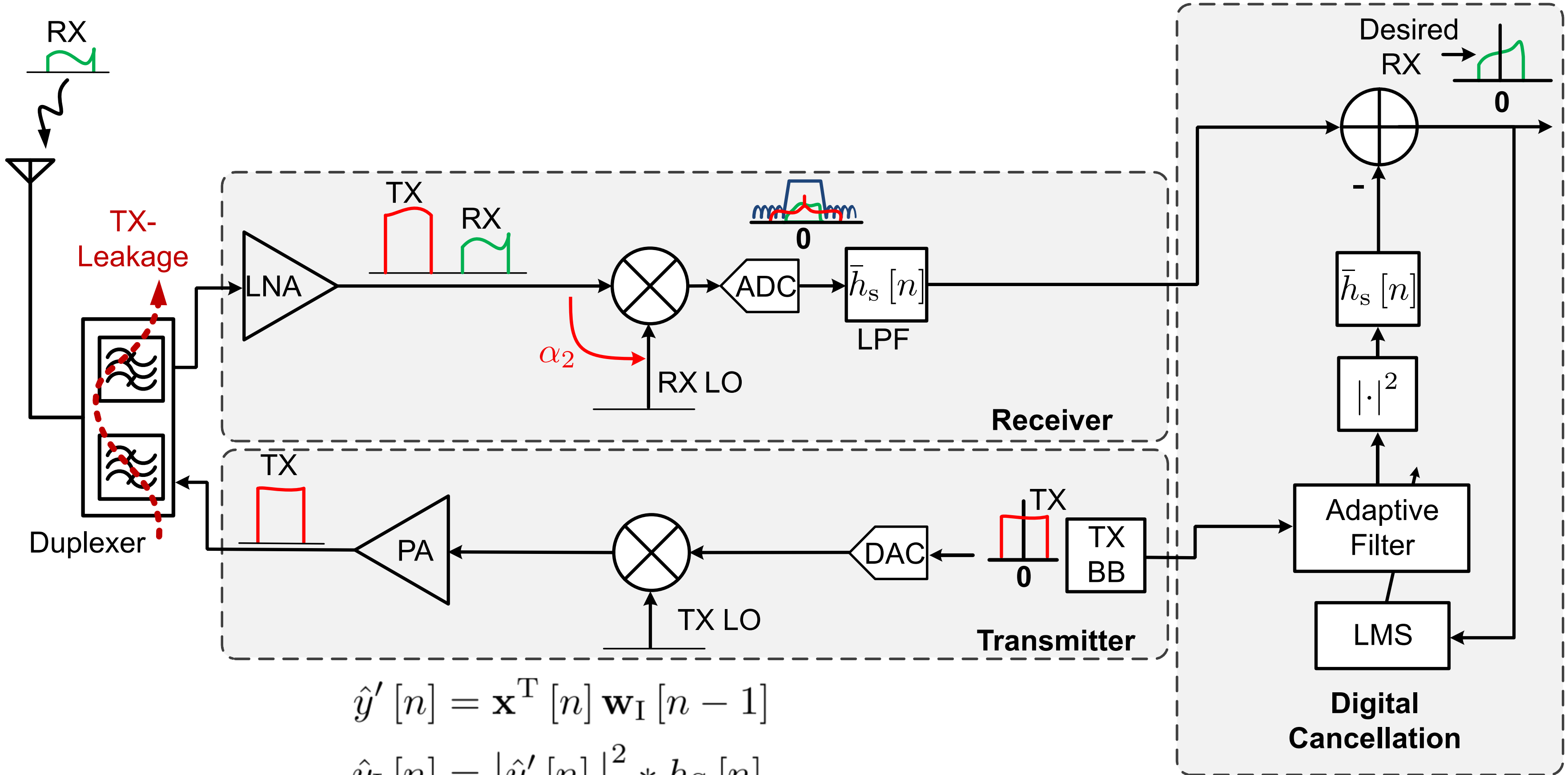


Self Interference Cancellation in Mobile Phone Transceiver RFICs (with Apple)



$$y_{\mathrm{BB}}^{\mathrm{Tot}}[n] = \underbrace{\frac{A_{\mathrm{LNA}}}{2} y_{\mathrm{BB}}^{\mathrm{Rx}}[n] * \bar{h}_{\mathrm{s}}[n]}_{\text{wanted Rx signal}} + \underbrace{\frac{\alpha_2^{\mathrm{I}} + j\alpha_2^{\mathrm{Q}}}{2} \left| A_{\mathrm{LNA}} A_{\mathrm{PA}} x_{\mathrm{BB}}[n] * h_{\mathrm{BB}}^{\mathrm{TxL}}[n] \right|^2 * \bar{h}_{\mathrm{s}}[n]}_{\text{interference}} + \underbrace{\frac{A_{\mathrm{LNA}}}{2} v_{\mathrm{BB}}[n] * \bar{h}_{\mathrm{s}}[n]}_{\text{noise}}$$

Self Interference Cancellation in Mobile Phone Transceiver RFICs (with Apple)



$$\hat{y}'[n] = \mathbf{x}^{\mathrm{T}}[n]\,\mathbf{w}_{\mathrm{I}}[n-1]$$

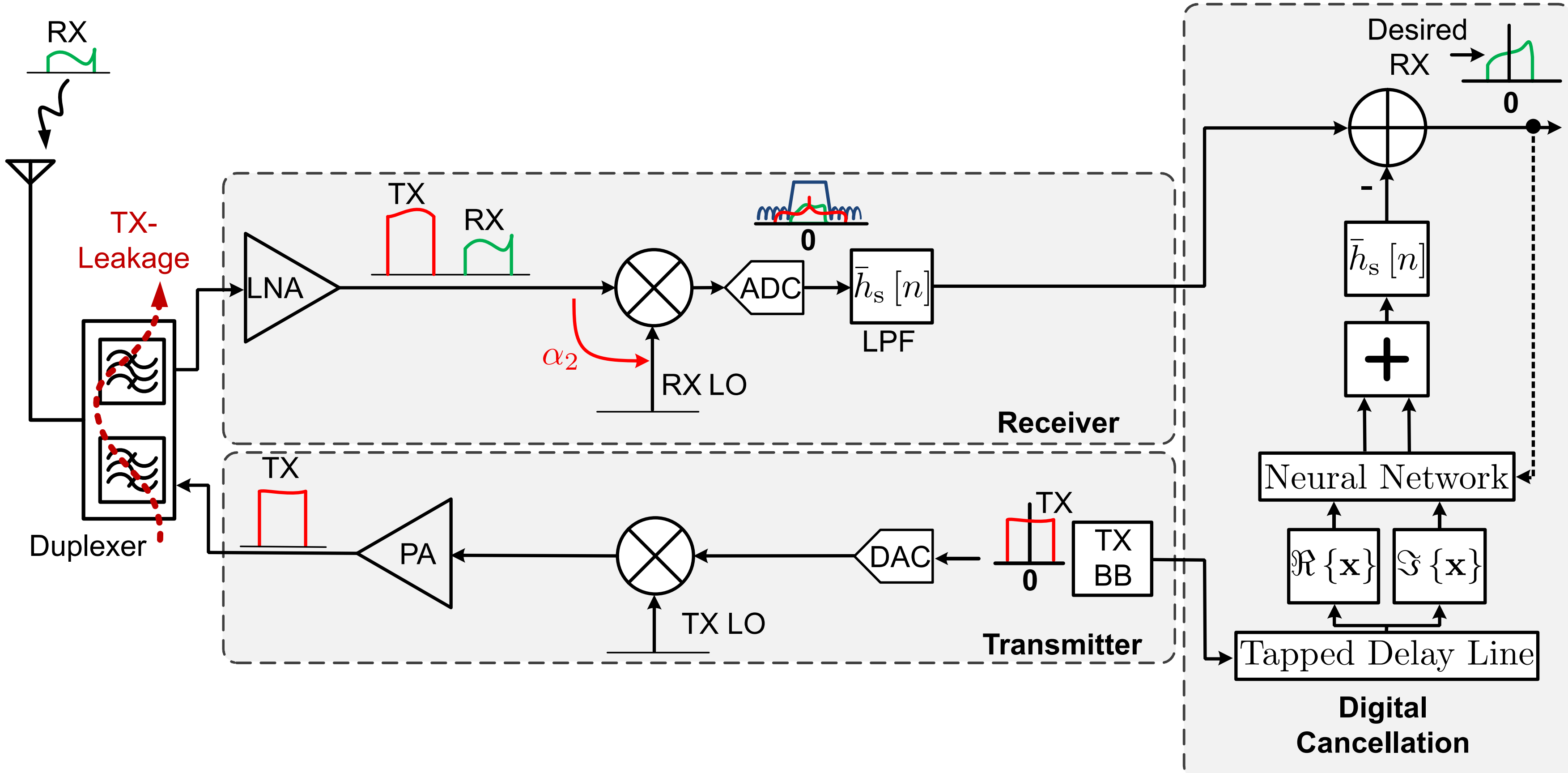
$$\hat{y}_{\mathrm{I}}[n] = \left|\hat{y}'[n]\right|^2 * h_{\mathrm{S}}[n]$$

$$\hat{y}_{\mathrm{AC,I}}[n] = a\hat{y}_{\mathrm{AC,I}}[n-1] + \hat{y}_{\mathrm{I}}[n] - \hat{y}_{\mathrm{I}}[n-1]$$

$$e_{\mathrm{AC,I}}[n] = d_{\mathrm{AC,I}}[n] - \hat{y}_{\mathrm{AC,I}}[n]$$

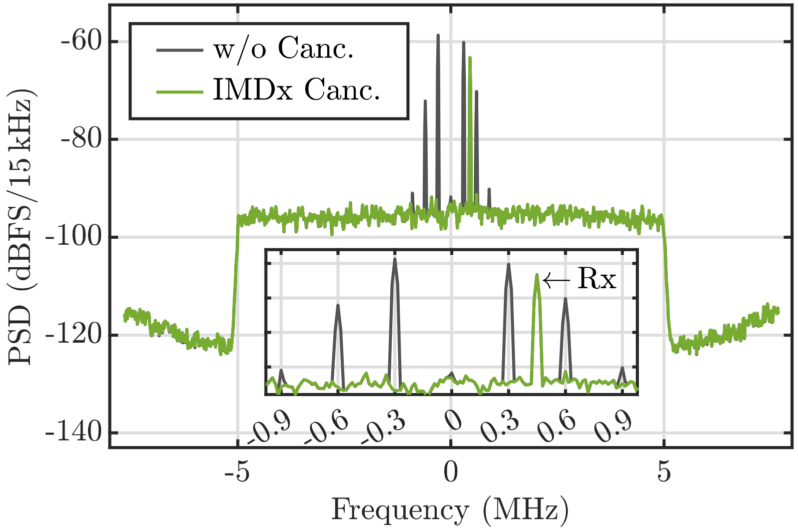
$$\mathbf{w}_{\mathrm{I}}[n] = \mathbf{w}_{\mathrm{I}}[n-1] + \frac{\mu e_{\mathrm{AC,I}}[n]\left(\hat{y}'[n]\,\mathbf{x}^*[n]\right) * h_{\mathrm{S}}[n]}{\epsilon + \left(\left|\hat{y}'[n]\right|^2 \mathbf{x}^{\mathrm{H}}[n]\,\mathbf{x}[n]\right) * h_{\mathrm{S}}[n]}$$

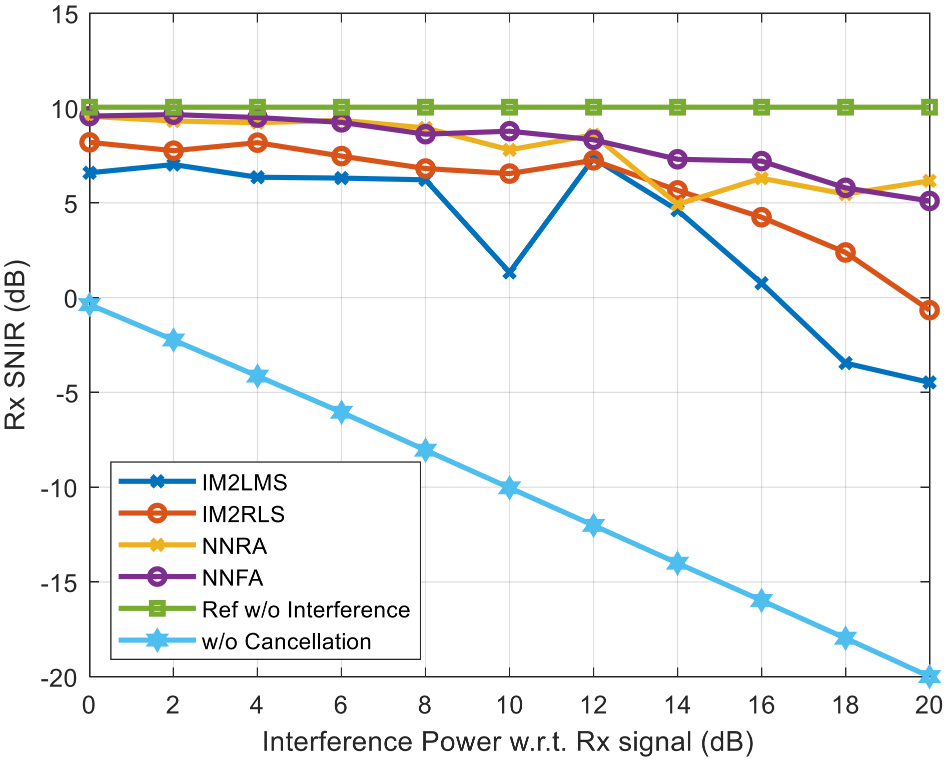
Self Interference Cancellation in Mobile Phone Transceiver RFICs (with Apple)



Self Interference Cancellation in Mobile Phone Transceiver RFICs (with Apple)

- Performance Results

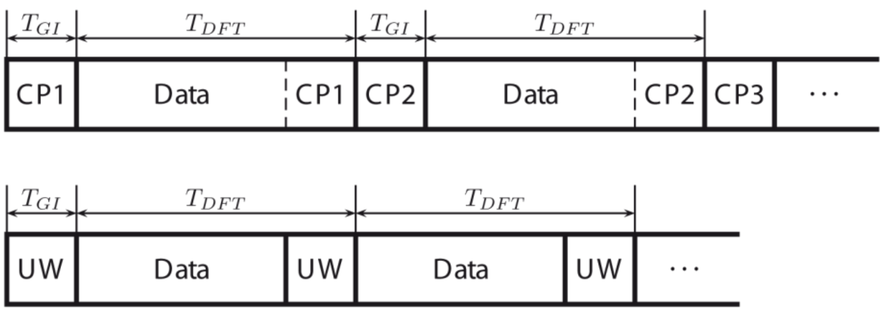




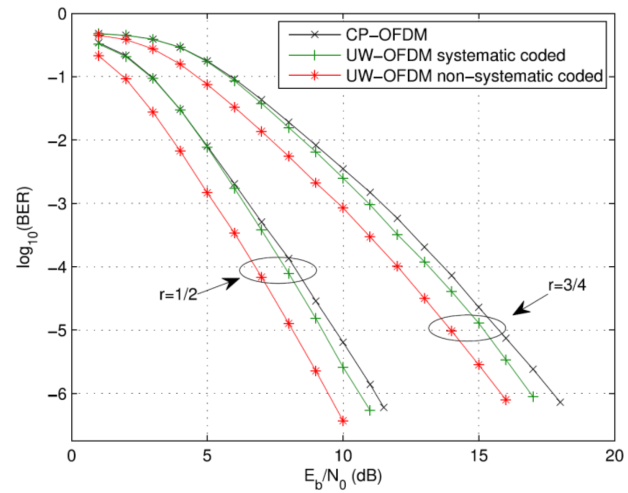
Source: A. Gebhard, C. Motz, R. S. Kanumalli, H. Pretl, and M. Huemer, "Nonlinear least-mean-squares type algorithm for second-order interference cancellation in LTE-A RF transceivers," in 2017 51st Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, USA, 2017, pp. 802–807.

Basic Research
Unique Word OFDM (FWF)

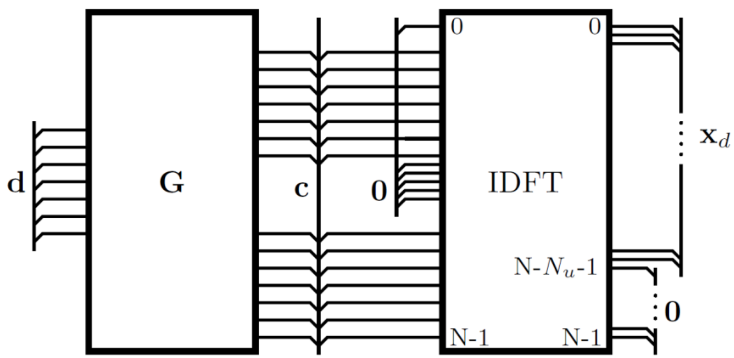
- Novel signaling scheme for digital communications



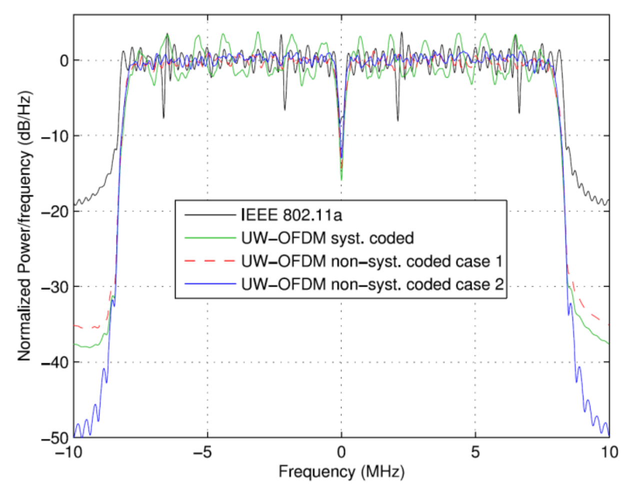
Bit error performance



Symbol generation



Welch Spectrum



Further Research Topics

- Parameter estimation (with voestalpine)
- Non-linear adaptive filters
- NN-based data estimation for wireless systems (with SAL)
- Joint communications and sensing
- Feature extraction of ECG signals (with KUK Linz)
- HW architectures for DSP
- …

- Diploma theses in co-operation with HTLs possible and welcome!

THANK YOU!